Integrating ANSYS with Modern Numerical Optimization Technology – Part I : Conjugate Feasible Direction Method

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1. Introduction

The Feasible Direction Method (FDM) was first proposed by Zoutendijk [1]. Later Vanderplaats et al. [2] implemented the algorithm into a computer code, and applied it on design optimization of space truss structures. Since then, FDM has been one of the most popular methods for finite element analysis based structural optimization. The concept of maintaining the search inside the feasible domain is found practical for engineering application. For decades, researchers have been applying this method on different engineering optimization problems. Some researchers also adopted this concept in the Sequential Quadratic Programming approach, and developed the Feasible Sequential Quadratic Programming (FSQP) approach [3].

Belegundu et at [4] used the interior point method to solve the linear programming sub-problem in FDM, and reported improvement on the efficiency. Some researchers also focus on improving the convergence properties and the generalization of the problem formulation [5,6].

During the last few years, there has been publications indicating that the conventional FDM could have problems dealing with certain structural optimization problems. The author proposed the cascade optimization approach [7], which was a combination of different algorithms. While the cascade approach successfully solved these problems, it did require restart from several different optimizers, and the computational effort could be enormous. There was no clear guidelines or rules for the combination of different optimizers either.

This research focuses on modifying the traditional FDM to integrate with ANSYS and achieve better convergence and efficiency properties. It is shown that, the modified algorithm can successfully solve the previously published problems which was not possible for traditional FDM. Two numerical examples were solved, and the results were compared with the existing publication.

2. The Conjugate Feasible Direction Method

The Method of Feasible Direction can be formulated as [1,2]

$$\begin{aligned} &\textbf{Maximize} && \beta \\ &\textbf{Subject to} && \nabla \mathbf{F}(\mathbf{X}^k)^T \cdot \mathbf{S}^k + \beta \le 0 \\ & && \nabla g_j(\mathbf{X}^k) \cdot \mathbf{S}^k + \theta_j \cdot \beta \le 0 \qquad j \in J \\ & && \mathbf{S}^{k^T} \cdot \mathbf{S}^k \le 1 \end{aligned} \tag{1}$$

Where J is the set of currently active constraints which $g_j(\mathbf{X}) = 0$.

This formulation is suitable for the case that the design is inside the feasible domain. For the case that the point is infeasible, it needs to be modified as

$$\begin{aligned} &\textbf{Maximize} && \nabla \mathbf{F}(\mathbf{X}^k)^T \cdot \mathbf{S}^k + \Phi \cdot \beta \\ &\textbf{Subject to} && \nabla g_j(\mathbf{X}^k)^T \cdot \mathbf{S}^k + \theta_j \cdot \beta \le 0 \qquad j \in J \\ & && \mathbf{S}^{k^T} \cdot \mathbf{S}^k \le 1 \end{aligned} \tag{2}$$

Where Φ is a very large number. The equations (1) and (2) are for calculating the Feasible Usable Direction, and one-dimensional line search is then performed to further improve the design, such that

Shen-Yeh Chen, Ph.D.
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$$\mathbf{X}^{k+1} = \mathbf{X}^k + \alpha \cdot \mathbf{S}^k \quad (3)$$

Define the Conjugate Feasible Direction (without detailed proof) as

$$\mathbf{S}_c^{k+1} = \mathbf{S}^{k+1} + \mu \cdot \mathbf{S}^k \quad (4)$$

If both iterations *k* and *k+1* are in the infeasible domain

$$\mu = \frac{i}{n} \cdot \frac{\mathbf{S}^{k+1^T} \cdot \mathbf{S}^{k+1}}{\mathbf{S}^{k^T} \cdot \mathbf{S}^k} \quad (5)$$

Otherwise

$$\mu = 0 \quad (6)$$

Here *i* is the number of consequent infeasible designs points, *n* is maximum number of conjugate gradient modification, which is set to the number of design variables. When *i* is greater then *n*, the counter is set to zero and the procedure is restarted. Or, whenever the design reaches a feasible point, the procedure is also restarted. $\mathbf{S}^{k+1}$ is calculated by equations (1) and (2), with the modified gradient of the objective function as

$$\nabla \mathbf{F}(\mathbf{X}^{k+1}) = \nabla \mathbf{F}(\mathbf{X}^{k+1}) + \frac{\nabla \mathbf{F}(\mathbf{X}^{k+1})^T \cdot \nabla \mathbf{F}(\mathbf{X}^{k+1})}{\nabla \mathbf{F}(\mathbf{X}^{k})^T \cdot \nabla \mathbf{F}(\mathbf{X}^{k})} \cdot \nabla \mathbf{F}(\mathbf{X}^k) \quad (7)$$

The gradient of the objective function is modified through all the design iterations, and is restarted every *n* iterations.

3. Stopping Criteria

In Vanderplaats's implementation of Feasible Direction Method [8], the problem was considered to be infeasible (without feasible solution) when certain number of consequent iterations failed to reach a feasible design. In this paper, we add some proprietary formulation to better judge the convergence of the iteration. This has been proved to be quite efficient and effective in the examples we have tested. Some of the examples are listed in this paper.

4. Numerical Examples

The above mentioned algorithm was implemented and integrated with the commercial finite element analysis package ANSYS. Two examples were solved to test the new algorithm. Both examples were from the same publication [7].

4.1 Example 1

This is the test example 2 in the previous publication. Figure 1 shows the configuration of the two-dimensional truss structure. The structure is fixed to the ground at nodes 1 and 6.

There are two load cases in this problem, which are listed in Table 1. In additional to the loading and the self-weight of the structure, nodes 2 and 4 are applied with point mass of weight 0.1943 pounds, and nodes 3 and 5 with point mass of 0.34974 pounds for both load cases. The elastic modulus is 10000 ksi, the Poisson's ratio is 0.3 and the density is 0.1 pound per cubic inch.

Load Case	Node ID	Direction	Value (pounds)
1	2	X	60000
1	2	Y	120000
1	3	X	60000
1	3	Y	60000
1	4	X	17500
1	4	Y	12500
1	5	X	17500
1	5	Y	12500
2	2	Y	-50000
2	3	Y	-25000
2	4	Y	-37500
2	5	Y	-75000

Table 1 Two Load Cases of Example 1

There are ten design variables in this problem, with each one assigned to the cross-section area of one individual member. All design variables have lower bounds of 0.01 square inches, 10000 square inches as the upper bounds, and 1.0 square inch as the initial values. The design constraints are shown in Table 2 for both load cases. The objective is to minimize the total volume of the structure.

Constraint Type	Applied Entity	Entity ID	Lower Bound	Upper Bound
Compressive Stress	Element	All	N/A	10000 (psi)
Tensile Stress	Element	All	N/A	10000 (psi)
Shear Stress	Element	All	N/A	10000 (psi)
Displacement (Y)	Node	3,4	N/A	2.2 inches
Frequency	Mode	1	26	N/A

Table 2 Design Constraints for Example 1

The design converges after thirty (30) iterations, with the final volume of 32692.62 cubic inches. The final design variables are listed in Table 3. In the original publication, the Feasible Direction Method failed to reach a feasible design.

DV	Element	Value (inch^2)
1	1	56.39
2	2	20.04
3	3	2.08
4	4	5.07
5	5	39.30
6	6	3.18
7	7	27.07
8	8	21.53
9	9	17.65
10	10	6.31

Table 3 Final Design Variables of Example 1

4.2 Example 2

This is the test example 9 in the previous publication. Figure 2 shows the configuration of the two-dimensional truss structure with nodes 1 and 6 fixed to the ground.

Table 4 lists the two load cases in this problem. Nodes 2 and 4 are applied with point mass of weight 0.1943 pounds, and nodes 3 and 5 with point mass of 0.34974 pounds for both load cases, in additional to the loading and the self-weight of the structure. The elastic modulus is 10000 ksi, the Poisson's ratio is 0.3 and the density is 0.1 pounds per cubic inch.

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Load Case	Node ID	Direction	Value (pounds)
1	2	X	60000
1	2	Y	120000
1	3	X	60000
1	3	Y	60000
1	4	X	17500
1	4	Y	12500
1	5	X	17500
1	5	Y	12500
2	2	Y	-50000
2	3	Y	-25000
2	4	Y	-37500
2	5	Y	-75000

Table 4 Two Load Cases of Example 2

There are ten design variables in this problem, with each one assigned to the cross-section area of one individual member. All design variables have lower bounds of 0.01 square inches, 10000 square inches as the upper bounds, and 1.0 square inch as the initial values. The design constraints are shown in Table 5 for both load cases. The objective is to minimize the total volume of the structure.

Constraint Type	Applied Entity	Entity ID	Lower Bound	Upper Bound
Compressive Stress	Element	All	N/A	10000 (psi)
Tensile Stress	Element	All	N/A	10000 (psi)
Shear Stress	Element	All	N/A	10000 (psi)
Displacement (Y)	Node	3,4	N/A	2.2 inches
Frequency	Mode	1	26	N/A

Table 5 Design Constraints for Example 2

The design converges after fifty-three (53) iterations, with the final volume of 52436.20 cubic inches. The final design variables are listed in Table 6. In the original publication, the Feasible Direction Method failed to reach a feasible design.

Element	Value (inch^2)
1	33.58
2	11.49
3	1.05
4	2.82
5	24.73
6	0.66
7	18.08
8	19.63
9	4.01
10	8.68

Table 6 Final Design Variables of Example 2

5. Conclusion

Modification to the conventional Feasible Direction Method is discussed. Special attention was paid to re-use the original algorithm but improve the convergence properties. The examples show that the

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proposed modification is able to achieve the feasible solution which is not possible for the original algorithm. The optimizer was combined with the existing finite element analysis package, and applied on daily design task. Further investigation is currently ongoing for basic mathematical properties of the new algorithm.

6. References

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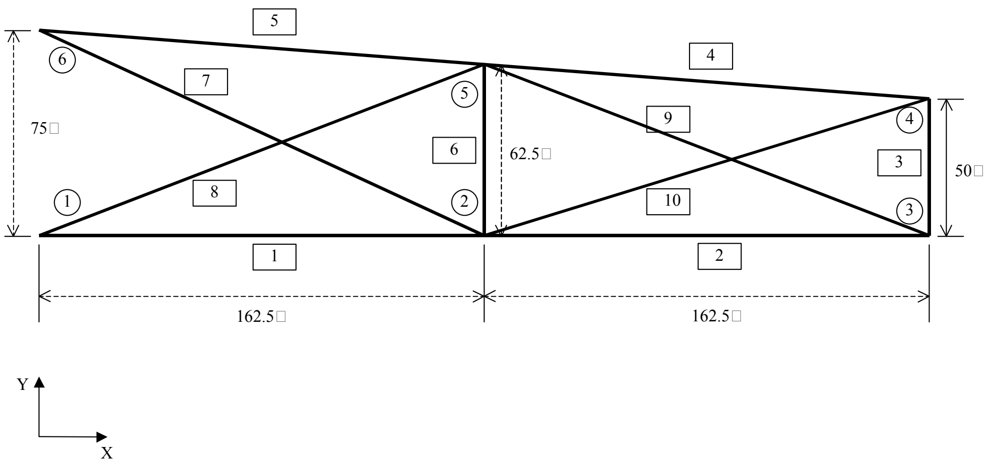

Figure 1 Structural Configuration of Example 1

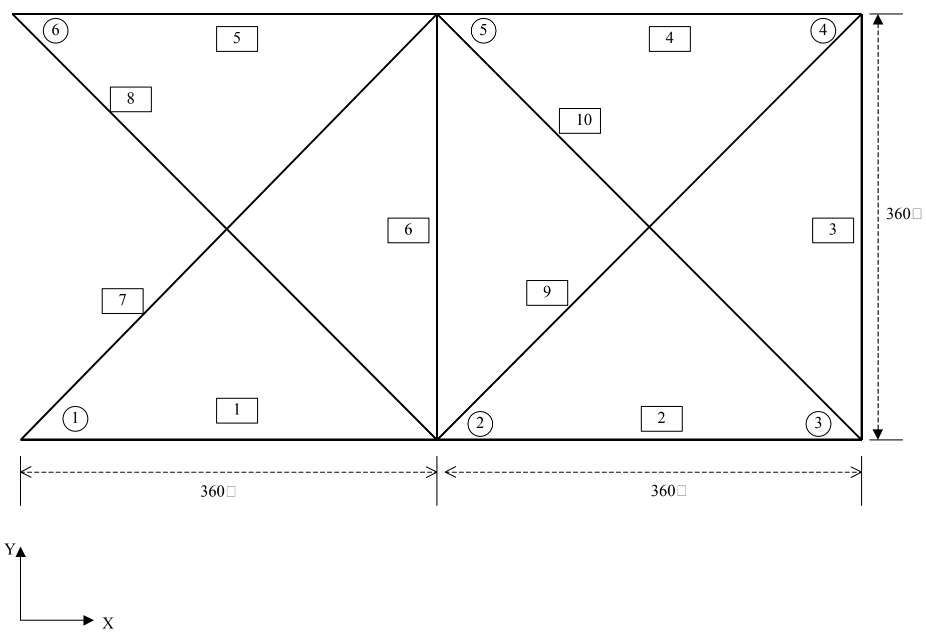

Figure 2 Structural Configuration of Example 2