Gradient-Based Structural and CFD Global Shape Optimization with SmartDO and the Response Smoothing Technology

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1. Abstract

A new general purpose design optimization commercial package SmartDO is applied to the structural and CFD global shape optimization problems. Using the new techniques called the Response Smoothing Technology implemented in SmartDO, it is possible to use the gradient-based approach, and still overcome the numerical noise caused by meshing and other discretization in FEA or FVM related application. This makes SmartDO able to perform global shape optimization with FEA and CFD applications in an efficient and systematic way. One structural and one CFD industry examples are illustrated. The results are compared with the ones optimized by conventional gradient-based algorithms. The new approach shows significant improvement over the traditional one.

2. Keywords

Global Optimization, Shape Optimization, Structural Optimization, CFD Optimization

3. Introduction

Structural shape optimization methodologies are fairly mature. However, one of the most important issues in structural shape optimization is still how to overcome the numerical noise cause by the meshing uncertainty of discretization approaches, like finite element analysis or finite volume methods.

Many approaches have been proposed to deal with the problems mentioned above, either directly or indirectly. The first approach is to control the meshing, so that the mesh pattern remains the same before and after the design changes. Some researchers uses the nodal coordinates as the design variables directly [1-2], or adding connections and constraints between nodes [3-4]. Others use the controlling parameters in CAD or geometric model as design variables, and relate the nodal coordinates with the blending interpolation or parameter of the geometry [5-9].

Two methods of mesh controlling seem to be popular in the commercial structural optimization packages today. The first method is called the fictitious load method or the Natural Shape Function approach [10]. An improved version of this approach, called the Hybrid Natural Approach [11], replaces fictitious load with displacements, and couples with geometry parameters to simplify the process of deciding design variables. The second method uses domain elements or super elements to define the perturbed mesh domain with a few controlling key nodes [12], which does not need the parametric information from the CAD system. This approach requires the original mesh to be divided into several domain elements. A new approaches called the Contour Natural Shape Function recently proposed by Chen et. al. use the so-called contour line to rebuild CAD-like controlling net outside the existing mesh, without any parametric information from the CAD, and don’t need to split the mesh into different domain either [13]. A thorough literature survey regarding the mathematical model shape changes is also conducted in the same paper [13]. These methods are also referred as “mesh-morphing”.

The second approach tries to focus on the optimization solver and algorithms. While the statistics-based approaches like genetic algorithms, has been applied on global optimization problems with certain reliability and stability [14,15], there have been very few researches devoted in the gradient-based approach to deal with the noise phenomena raised from the CAE-based problem. Chen [16] uses the Lagrange interpolation together with the Design of Experiment to construct the global response surfaces, and use genetic algorithms to search the global minimum. The approach was successfully applied to solve the optimization of ballistic shield with explicit dynamic finite element analysis. Snyman [17] uses central finite difference to calculate the gradient of the objective function. This approach is able to overcome the noise in the objective function. However, there is no CAE-based example in the paper.

This paper deal with the numerical noise observed in the CAE-based or FEA-based optimization problems using the Response Smoothing Technology implemented in the commercial package SmartDO [18]. Two examples, one for the structural shape optimization, and one for the CFD-based shape optimization, will be illustrated later.

4. The Response Smoothing Technology and Implementation

4.1. Numerical Noise
In the CAE-based or FEA-based application of design optimization, the design-analysis loop usually looks like Figure 1. Several situations can cause the numerical noise. One of the sources is the meshing. When the mesh is created in free form, the mesh pattern is likely to change before and after the design changes (for example, from a mesh with n nodes into a mesh with n+m nodes).

![Figure 1 The Design-Analysis Loop](image)

When the numerical noise exists in the design-analysis loop, the relationship between the model response and the design variables changes appears similar to Figure 2. This will create many artificial local minima, even the problem itself is actually convex and has only one local minimum. For methods that use the mesh-morphing, the design-analysis loop becomes Figure 3. This can eliminate the numerical noise caused by mesh, but there are still some other sources of noise.

![Figure 2 The Numerical Noise Appears in the CAE-Based Optimization](image)
4.2. The Response Smoothing Technology

Generally, the problems of nonlinear programming can be formulated as equation (1)

\[
\begin{align*}
\text{find} & \quad x = \{x_1, x_2, \ldots, x_{NDV}\} & \text{Design Variables} \\
\text{to minimize} & \quad f(x) & \text{Objective Function} \\
\text{subjected to} & \quad G_i(x) = \frac{g_i(x)}{g_i^0} - 1 \leq 0 \quad i = 1, \ldots, NINEQC & \text{(Inequality) Constraints} \\
& \quad x_k^L \leq x_k \leq x_k^U \quad k = 1, \ldots, NDV & \text{Lower/Upper Bounds}
\end{align*}
\]

Most of the gradient-based solution algorithms require equation (1) to be smooth and well-behaved, and can only solve the local optimum. Since the numerical noise is considered in this paper, the solver will have to search for global minimum. In order to achieve this goal, equation (1) is modified as

\[
\begin{align*}
\text{find} & \quad x = \{x_1, x_2, \ldots, x_{NDV}\} & \text{Design Variables} \\
\text{to minimize} & \quad w \cdot f(x) + \Phi & \text{Objective Function} \\
\text{subjected to} & \quad G_i'(x) = \frac{g_i(x)}{g_i^0} - 1 - \phi_i \leq 0 \\
& \quad i = 1, \ldots, NINEQC & \text{(Inequality) Constraints} \\
& \quad x_k^L \leq x_k \leq x_k^U \quad k = 1, \ldots, NDV & \text{Lower/Upper Bounds}
\end{align*}
\]

Here \(w\), \(\Phi\) and \(\phi_i\) are proprietary formulations which take into account the values of objective function and constraints from the design history. The new formulation is then solved by usual nonlinear programming techniques. The purpose of these three additional functions is to push the design point out of the local minimum, and avoid the search path in the design history. Figure 4 shows the schematics of how the search path will look like.
4.3. The Computer Program SmartDO
The Response Smoothing Technology in the previous section is implemented in the commercial package SmartDO. SmartDO uses the standard Tcl/Tk shell as the user interface. The users communicate SmartDO and external packages through Tcl/Tk script. Because the optimizer is embedded into Tcl/Tk script, SmartDO can utilize full functionality of Tcl/Tk. Therefore it is powerful for coupling with any external CAE packages. The basic architecture of SmartDO is shown in Figure 5.

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5. Numerical Examples

5.1. Example for Structural Shape Optimization
Figure 6 shows the front view, side view and isotropic view of the original design a giant hook. The hook was use on some critical mission for lifting materials. The totally height of the hook is about 1.8 m, and the total weight about 700 kg. The hook is made of structural steel.
It is very critical to reduce the weight of the hook, otherwise it may be difficult to find the lifting facilities able to provide the capability. With the rising expense of steel globally, the cost of the material is another issue. It will be also dangerous to operate a hook with such a huge weight.

The goal of the design is to minimize the weight of the hook as much as possible, while the maximum equivalent stress should remain the same or be lower. The analysis will have to be performed by the customer’s existing commercial package. The parametric model is built for the commercial package, but only free meshing is possible. Totally twenty-three design variables are used as shown in Figure 7, which link to the coordinates of certain points, and the dimension at certain location. The model is built in three-dimensional geometry.

There are at least two sources of numerical noise in this problem, namely, taking maximum equivalent stress of the model and free meshing. Also, from experience, local minimum is expected.
SmartDO was used to solve this problem, with the Response Smoothing Technology option turned on. Figure 8 shows the front view and side view of the optimal design. For comparison purpose, Figure 9 shows the original design (left), the design optimized by the traditional Methods of Feasible Directions [19] (MFD, center), and the design optimized by SmartDO with the Response Smoothing Technology (RST, right). The design in the center only has very minor changes in each design variable and dimension (about less than 1%), and the reduction in weight is only 0.09% (with maximum stress satisfying the stress constraint). However, the design by SmartDO reduces the weight by 20%, and the maximum stress is a little less than the original design. From the result it is obvious that the Method of Feasible Directions was trapped in the local minimum due to numerical noise, and the Response Smoothing Technology was able to escape from the local minimum.

Figure 8 Different View of the Optimum Design by SmartDO

Figure 9 Original Design (Left) and Optimal Design by Method of Feasible Directions (Center) and Response Smoothing Technology (Right)

5.2. Example for CFD Shape Optimization
This example is for the design of an industrial ejector. An industrial ejector is a facility used to mix two flows and compress them into higher pressure. Figure 10 shows the two-dimensional design drawing of an ejector. The primary flow enters the nozzle with specific pressure, and is ejected into the flow path. The primary flow will usually reach supersonic speed when passes through the throat of the nozzle. After existing the nozzle and entering the mixing zone, the primary flow is expanded, which will create negative (suction) pressure, and cause the secondary flow to flow and mix in. Finally
the mixed flow will exist the ejector with a (hopefully) higher pressure.

Figure 10 The Two-Dimensional Drawing of an Ejector

The ejector is simplified and modeled in a commercial package as shown in Figure 11. The model is axial symmetric, with the symmetric axis noted as Figure 11. The boundary P-in1 presents the inlet of primary flow, and the P-in2 as the inlet of secondary flow. P-out is the outlet of the mixed flow, and all other boundaries are modeled as static walls. The pressure and temperature of P-in1 and P-out is given, and the flow rate and temperature of P-in2 is also given. The design variables and the geometry are defined in Figure 12. There are totally 13 design variables, selected as D2, D4, D5, D6, D10, D11, D12 L4, L9, (L10-L9), (L11-L10), (L5-L11) and (L6-L5). Each design variable has its lower and upper bound.

Figure 11 Simplified CFD Model of the Ejector
For the current example, the goal is to maximize the compression ratio, which is defined as

\[ C_r = \frac{\text{Pressure of } P_{-\text{out}}}{\text{Pressure of } P_{-\text{in}2}} \]  

(3)

The constraints are mostly geometric constraints, which limit the range and relationship between the design variables. From the initial sensitivity study by manual adjusting the design variables, it was found that the problem had very serious problems of numerical noise. There are two reasons for the numerical noise. First, the mesh is created in free form. Second, the pressure at P-in2 has to be integrated along the line, and then the average is taken.

Table 1 shows the CFD results of the original design, and those optimized by Response Smoothing Technology (RST) and by the Method of Feasible Directions (MFD). The result shows that, while the design optimized by RST can increase about 390% of the compression ratio compared to the original design, the one optimized by MFD has only 5% improvement. Table 2 shows the final design variables of all three configurations. It can be seen that for certain design variables, RST and MFD is going in the opposite direction of change. Also, most changes of design variables from MFD are very insignificant. This example has proved that RST can at least escape from local minimum in many situations.

Table 1 CFD Result for the Ejector Model, Original, Response Smoothing Technology (RST), and Method of Feasible Directions (MFD)

<table>
<thead>
<tr>
<th>model</th>
<th>P-in1</th>
<th>P-in2</th>
<th>P-out</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>957.68</td>
<td>1.96</td>
<td>4.00</td>
<td>2.04</td>
</tr>
<tr>
<td>RST</td>
<td>957.55</td>
<td>0.40</td>
<td>4.00</td>
<td>9.94</td>
</tr>
<tr>
<td>MFD</td>
<td>957.40</td>
<td>1.86</td>
<td>4.00</td>
<td>2.15</td>
</tr>
</tbody>
</table>
Table 2 Comparison of the Finally Design Variables for the Model, Original, Response Smoothing Technology (RST), and Method of Feasible Directions (MFD)

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>RST</th>
<th>MFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>5.30</td>
<td>5.06</td>
<td>5.41</td>
</tr>
<tr>
<td>D4</td>
<td>50.30</td>
<td>52.35</td>
<td>49.94</td>
</tr>
<tr>
<td>D5</td>
<td>80.30</td>
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<td>80.81</td>
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<tr>
<td>D6</td>
<td>201.47</td>
<td>202.02</td>
<td>201.47</td>
</tr>
<tr>
<td>D10</td>
<td>160.30</td>
<td>158.98</td>
<td>161.11</td>
</tr>
<tr>
<td>D11</td>
<td>135.30</td>
<td>131.35</td>
<td>135.02</td>
</tr>
<tr>
<td>D12</td>
<td>80.30</td>
<td>101.07</td>
<td>80.36</td>
</tr>
<tr>
<td>L4</td>
<td>280.96</td>
<td>312.80</td>
<td>281.05</td>
</tr>
<tr>
<td>L5</td>
<td>1385.92</td>
<td>1365.65</td>
<td>1386.13</td>
</tr>
<tr>
<td>L6</td>
<td>2195.92</td>
<td>2173.20</td>
<td>2196.25</td>
</tr>
<tr>
<td>L9</td>
<td>520.96</td>
<td>520.37</td>
<td>520.92</td>
</tr>
<tr>
<td>L10</td>
<td>640.96</td>
<td>640.77</td>
<td>641.28</td>
</tr>
<tr>
<td>L11</td>
<td>1225.92</td>
<td>1209.19</td>
<td>1226.23</td>
</tr>
</tbody>
</table>

6. Conclusions
A new numerical optimization approach, the Response Smoothing Technology, for dealing with numerical noise in CAE-based optimization is developed. It was applied on structural and CFD shape optimization. The examples have shown that, the new approach can at least escape from the local minimum of CAE-based application. And the reduction of the objective function values is significantly larger than the one optimized by the traditional methods. Since the algorithms were embedded into Tcl/Tk shell, the integration with other packages is flexible and easy.

7. Acknowledgement
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8. References
11. S.D. Rajan, S-W. Chin, and L. Gani, Toward a Practical Design Optimization Tool, Microcomputer in Civil